Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International GCSE

**Time** 2 hours

**Paper reference** **4PM1/01R**

# Further Pure Mathematics
## PAPER 1R

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**International GCSE in Further Pure Mathematics Formulae sheet**

## Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

## Series

### Arithmetic series

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### Geometric series

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

### Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

## Calculus

### Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

## Trigonometry

### Cosine rule

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** (a) On the grid below, draw the graph of the line with equation

(i) $4x + 5y = 20$ (ii) $3y - 4x = -12$

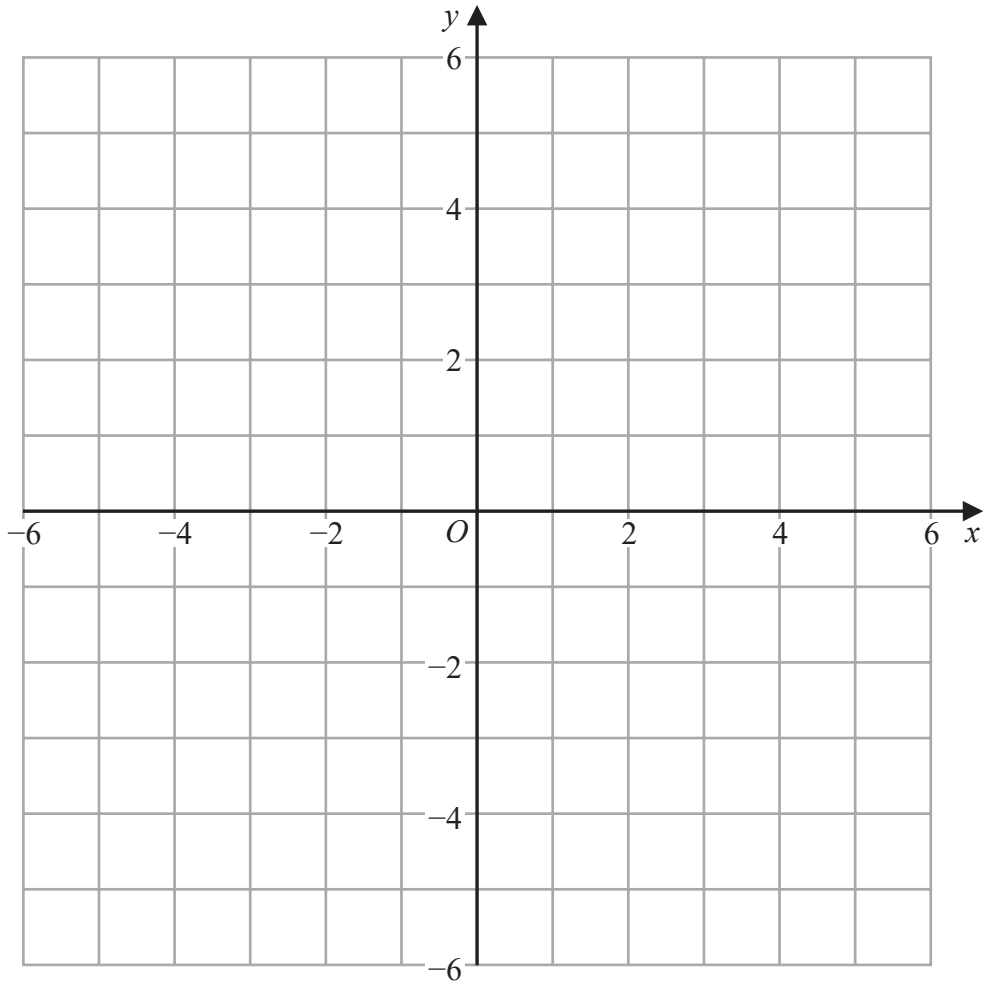
**(2)**

(b) Show, by shading on the grid, the region $R$ defined by the inequalities

$4x + 5y \leqslant 20$ $3y - 4x \geqslant -12$ $y \leqslant 3$ $x \geqslant 1$

Label the region $R$.

**(2)**




**Turn over for a spare grid if you need to redraw your graph.**

**Question 1 continued**

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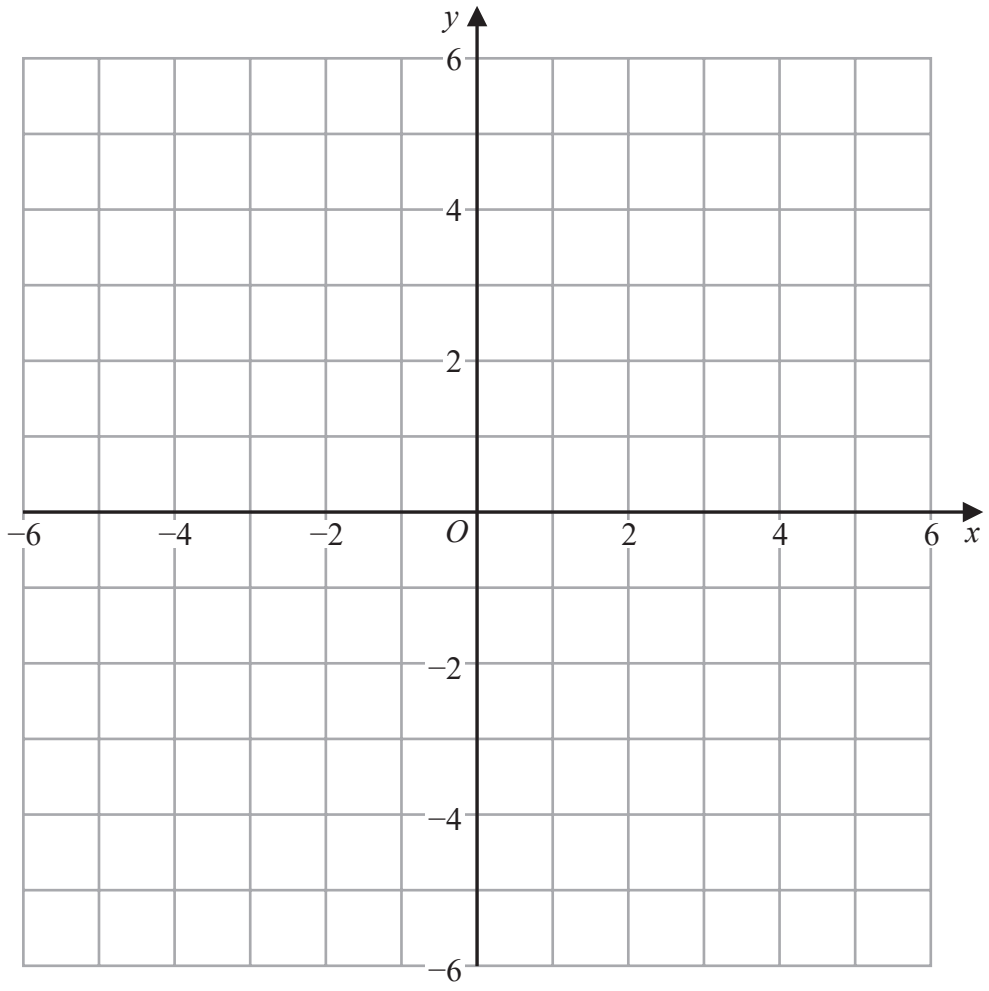
**Question 1 continued**

**Only use this grid if you need to redraw your graph.**




**(Total for Question 1 is 4 marks)**

**2** The $n$th term of an arithmetic series is $u_n$

Given that $u_5 = 46$ and that $u_{20} = 181$

(a) find

(i) the common difference of this series,

(ii) the first term of this series.

**(4)**

(b) Evaluate $\sum_{n=21}^{50} u_n$

**(4)**

**Question 2 continued**

**(Total for Question 2 is 8 marks)**

**3** The point $A$ has coordinates (1, 7) and the point $B$ has coordinates (9, 3)

The line $l$ is the perpendicular bisector of $AB$

(a) Find an equation of $l$

**(5)**

The line $l$ crosses the $x$-axis at the point $C$

(b) Find the area of the triangle $ABC$

**(5)**

**Question 3 continued**

**(Total for Question 3 is 10 marks)**

**4**

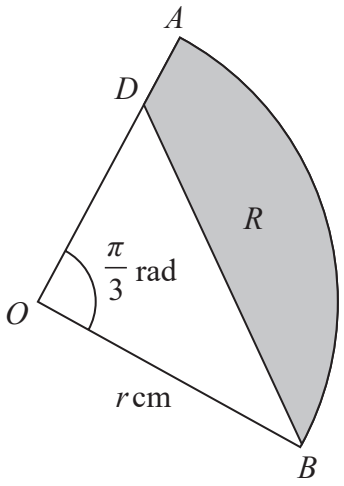



Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows sector $OAB$ of a circle with centre $O$ and radius $r$ cm.

The angle $AOB = \frac{\pi}{3}$ radians.

The point $D$ divides $OA$ in the ratio $3 : 1$

The area of the region $R$, shown shaded in Figure 1, is $79.5\,\text{cm}^2$

(a) Calculate the value, to 2 significant figures, of $r$

**(4)**

(b) Calculate the perimeter, in cm to 2 significant figures, of the region $R$

**(4)**

**Question 4 continued**

**(Total for Question 4 is 8 marks)**

**5**

$$f(x) = (1 + ax)^n \text{ where } n > 2$$

(a) Write down the expansion of $f(x)$ in ascending powers of $x$ up to and including the term in $x^3$
Give each coefficient in terms of $n$ and $a$

**(2)**

In the expansion of $f(x)$, the coefficient of $x$ is 8 and the coefficient of $x^2$ is 30

(b) Find the value of $n$ and the value of $a$

**(4)**

(c) Calculate the coefficient of $x^3$ in $f(x)$

**(2)**

**Question 5 continued**

**(Total for Question 5 is 8 marks)**

**6** The points $P$, $Q$, $R$ and $S$ are the vertices of a quadrilateral $PQRS$ such that

$$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} \qquad \overrightarrow{PR} -\mathbf{i} + 18\mathbf{j} \qquad \overrightarrow{PS} = -3\mathbf{i} + 15\mathbf{j}$$

(a) Show that $PQRS$ is a parallelogram.

**(4)**

(b) Find a unit vector parallel to $\overrightarrow{QS}$ as a simplified expression in terms of **i** and **j**

**(4)**

The point $T$ lies on $QS$ such that $QT : TS = 5 : 8$

(c) Find $\overrightarrow{PT}$ as a simplified expression in terms of **i** and **j**

**(2)**

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**Question 6 continued**

**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 10 marks)**

7

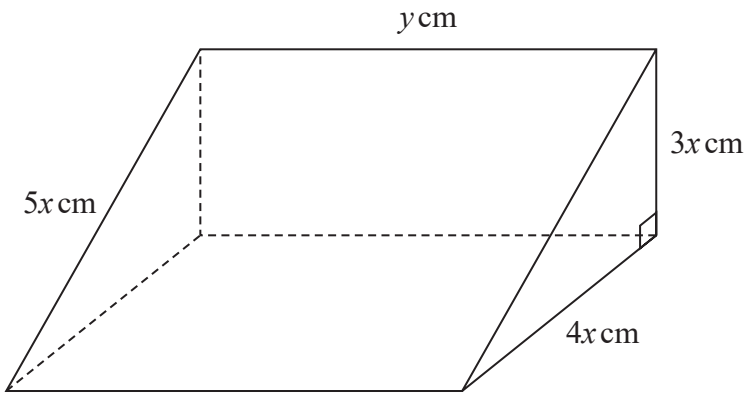


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**Figure 2**

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length $3x$ cm, $4x$ cm and $5x$ cm.

The length of the prism is $y$ cm.

The total surface area of the five faces of the prism is $144\,\text{cm}^2$

The volume of the prism is $V\,\text{cm}^3$

(a) Show that

$$V = 72x - 6x^3$$

**(5)**

Given that $x$ can vary,

(b) use calculus to find the value of $x$ for which $V$ is a maximum, justifying that this value gives a maximum value of $V$

**(4)**

(c) Find the maximum value of $V$

**(2)**

**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 11 marks)**

**8** The curve $C$ has equation $y = 2x^2 - \sin x$

The point $A$ on $C$ has $x$ coordinate $\pi$

Show that an equation of the normal to $C$ at the point $A$ is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0$$

**(8)**

**Question 8 continued**

**(Total for Question 8 is 8 marks)**

**9** (a) Complete the table of values for

$$y = \frac{x^3 + 4}{5 - x}$$

giving your answers to 2 decimal places.

| $x$ | −2 | −1.5 | −1 | −0.5 | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | −0.57 | 0.10 | 0.5 | | 0.8 | | 1.25 | | 4 |

**(2)**

(b) On the grid opposite, draw the graph of

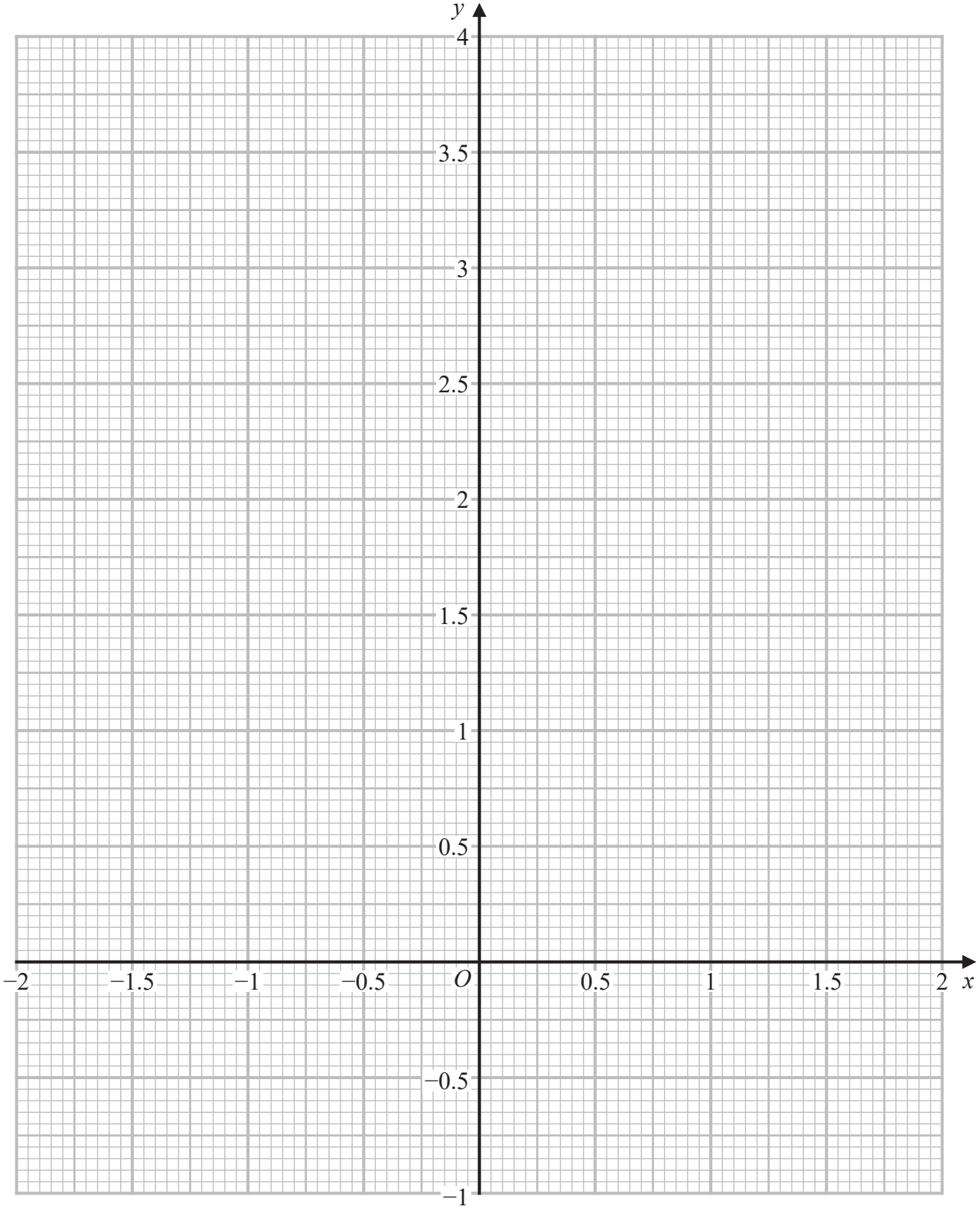
$$y = \frac{x^3 + 4}{5 - x} \qquad \text{for } -2 \leqslant x \leqslant 2$$

**(2)**

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $x^3 - x^2 + 8x - 11 = 0$ in the interval $-2 \leqslant x \leqslant 2$

**(5)**

**Question 9 continued**




**Turn over for a spare grid if you need to redraw your graph.**

**Question 9 continued**

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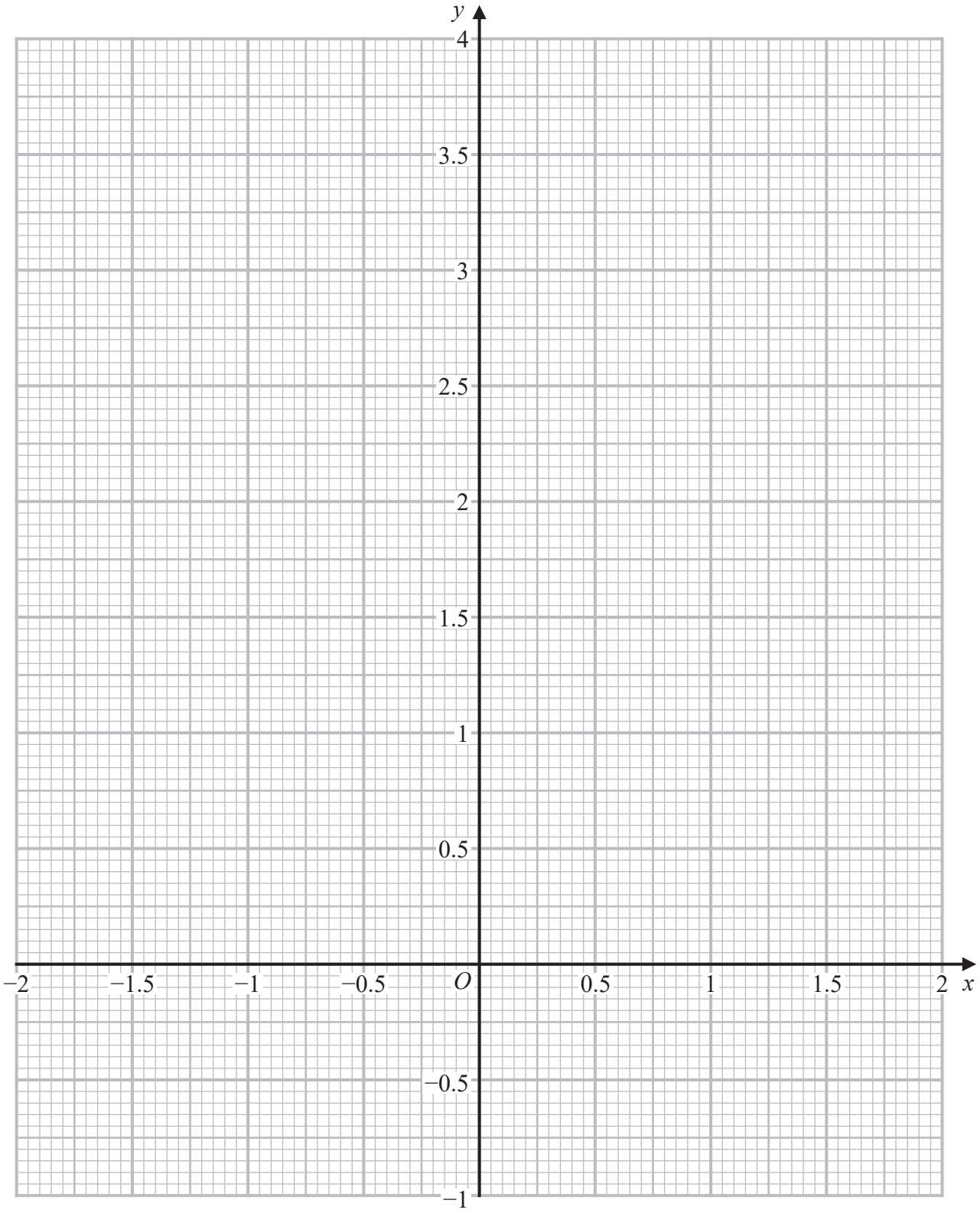
**Question 9 continued**

**Only use this grid if you need to redraw your graph.**




**(Total for Question 9 is 9 marks)**

**10** (a) Use the factor theorem to show that $(4x - 3)$ is a factor of

$$16x^3 + 11x - 15$$

**(2)**

(b) Using formulae given on page 2, show that

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1$

**(5)**

(c) Show that the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution $x = \cos \theta$

**(4)**

(d) Hence show that any solution of the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

is given by $\cos \theta = \frac{3}{4}$

**(4)**

**Question 10 continued**

**Question 10 continued**

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**Question 10 continued**

**(Total for Question 10 is 15 marks)**

**11**

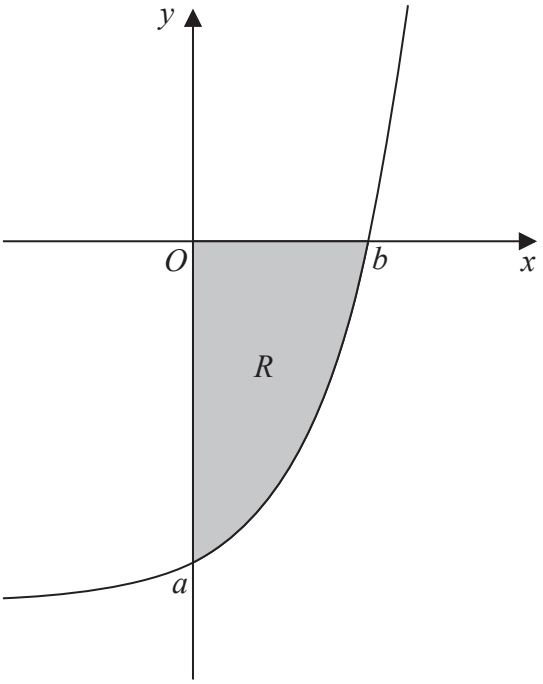


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**Figure 3**

The finite region $R$, shown shaded in Figure 3, is bounded by the curve with equation $y = e^{2x} - 9$ and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates $(0, a)$ and $(b, 0)$

(a) (i) Find the value of $a$

(ii) Show that $b = \ln 3$

**(3)**

The region $R$ is rotated through 360° about the $x$-axis.

(b) Use calculus to find the volume of the solid generated.

Give your answer in the form $\pi(p \ln 3 + q)$, where $p$ and $q$ are integers.

**(6)**

**Question 11 continued**

**Question 11 continued**

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**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 9 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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