Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Monday 20 January 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

Further Pure Mathematics

Paper 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

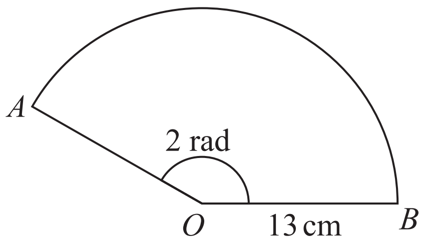

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector AOB of a circle with centre O.
The radius of the circle is 13 cm and angle $AOB = 2$ radians.

(a) Find the length of the arc AB.

(1)

(b) Find the area of the sector AOB.

(2)

(Total for Question 1 is 3 marks)

2

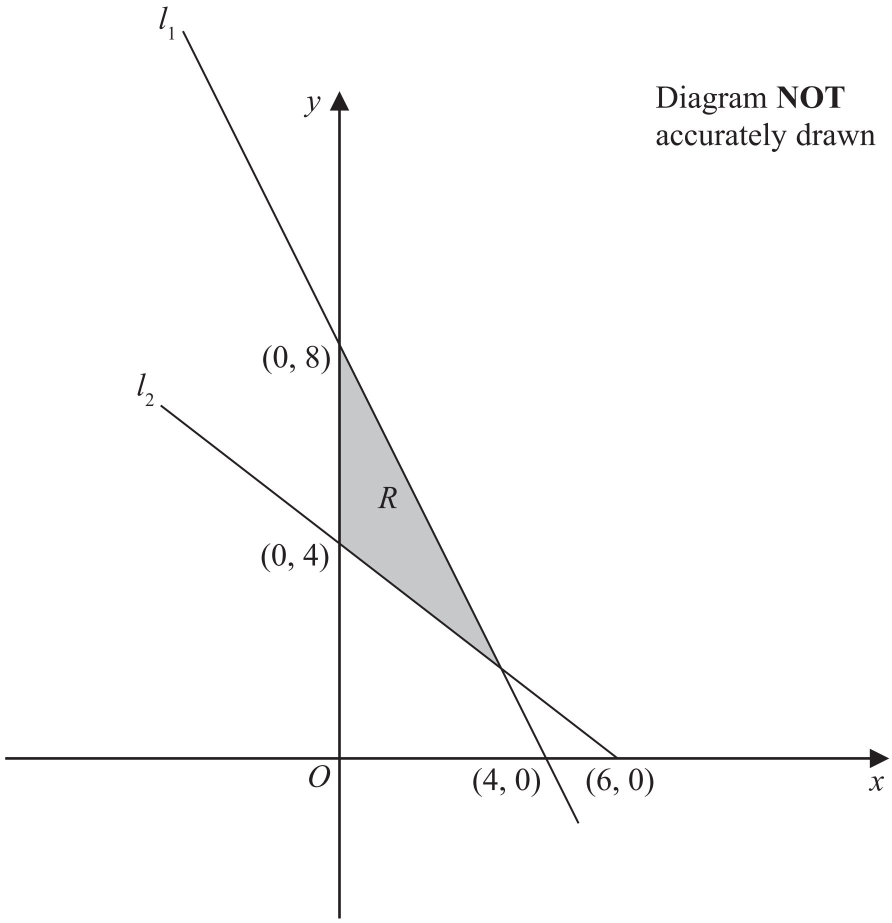

Figure 2

Figure 2 shows the shaded region R bounded by the line l_1, the line l_2 and the y-axis.

The points with coordinates (0, 8) and (4, 0) lie on l_1

The points with coordinates (0, 4) and (6, 0) lie on l_2

(a) Find, in the form $ax + by = c$, where a, b and c integers, an equation of

(i) l_1

(ii) l_2

(3)

(b) Hence write down three inequalities that define the region R.

(3)

Question 2 continued

(Total for Question 2 is 6 marks)

3 In triangle ABC, $AB = 11$ cm and $BC = 12$ cm.

The area of triangle $ABC = 33\text{ cm}^2$

Find, in cm to 3 significant figures, the two possible lengths of AC.

(5)

Question 3 continued

(Total for Question 3 is 5 marks)

4

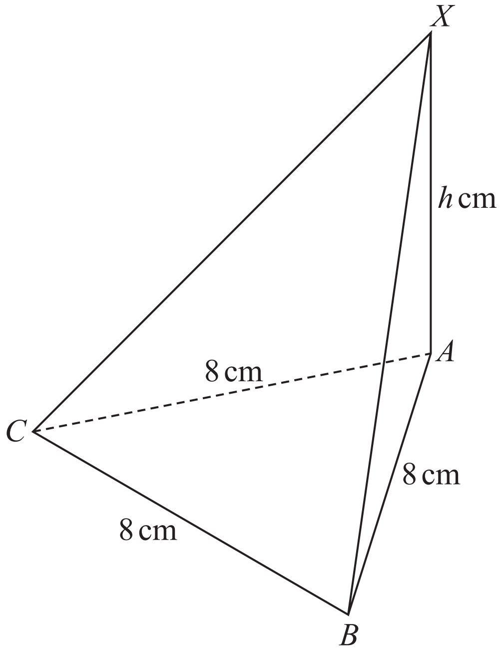

Figure 3

Figure 3 shows a triangular pyramid *ABCX*.
The base *ABC* of the pyramid is an equilateral triangle where $AB = BC = CA = 8$ cm.
The vertex *X* of the pyramid is such that *AX* is perpendicular to the base of the pyramid and $AX = h$ cm.

The volume of the pyramid is $48\sqrt{3}$ cm^3

(a) Show that $h = 9$

(3)

(b) Find, in degrees to one decimal place, the size of angle *BXC*.

(3)

(c) Find, in degrees to one decimal place, the size of the angle between the plane *BCX* and the base *ABC* of the pyramid.

(3)

Question 4 continued

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Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5 (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$

(2)

The roots of the equation $2x^2 + 3x + 6 = 0$ are α and β

Without solving the equation,

(b) find the value of $\alpha^3 + \beta^3$

(2)

(c) Show that $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$

(2)

(d) Form a quadratic equation with integer coefficients that has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$

(6)

Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 12 marks)

6

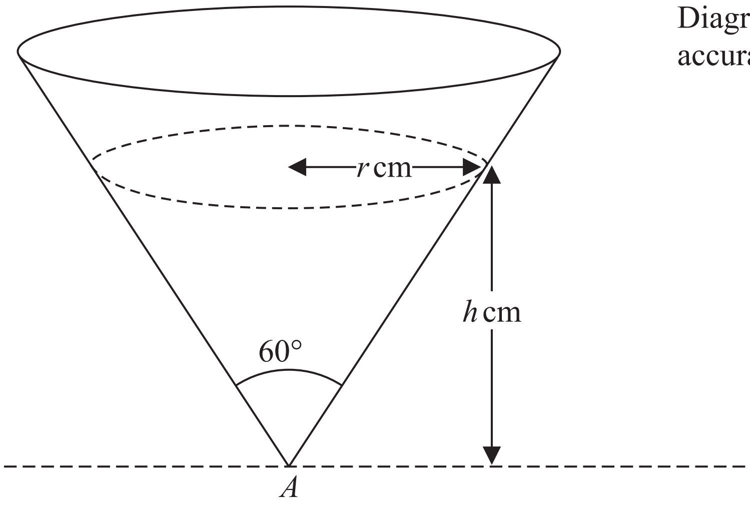

Figure 4

Figure 4 shows a hollow right circular cone fixed with its axis of symmetry vertical.

The cone is inverted and contains liquid, which is dripping out of a small hole at the vertex A of the cone at a constant rate of $0.9\,\text{cm}^3/\text{s}$.

At time t seconds after the liquid starts to drip from the cone, the height of the liquid is h cm above A. The volume of liquid in the cone at time t seconds is $V\,\text{cm}^3$

The vertical angle of the cone is 60°

(a) Show that $V = \frac{1}{9}\pi h^3$

(2)

(b) Find, in cm/s to 3 significant figures, the rate at which the height of the liquid is decreasing when the height of the liquid in the cone above the vertex is 1.2 cm.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 6 marks)

7 The geometric series G has first term a, common ratio r and nth term u_n

Given that $u_4 = e^{x+2}$ and that $u_7 = e^{\frac{2x+1}{2}}$

(a) show that $r = e^{-\frac{1}{2}}$

(3)

(b) Hence find a in terms of e and x.

(3)

Given that the sum to infinity of G can be written as $\dfrac{e^p}{e^{\frac{1}{2}} - 1}$

(c) find an expression for p in terms of x.

(3)

Given that $u_{18} > 1.6$ and that x is an integer,

(d) find the least value of x.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

8 (a) Write down the value of k such that $\sin 2A = k \sin A \cos A$

(1)

$$g(A) = 2 + 3\cos A - \sin A - 3\sin 2A - 2\cos^2 A$$

Given that $g(A)$ can be written in the form $(p\cos A - \sin A)(q - r\sin A)$ where p, q and r are integers,

(b) find the value of p, the value of q and the value of r.

(3)

(c) Hence solve, in radians to 3 significant figures where appropriate, the equation

$$g(2\theta) = 0 \quad \text{for} \quad 0 \leqslant \theta < \pi$$

(6)

Question 8 continued

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Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9 Given that $\dfrac{1}{(2-x)^3}$ can be written as $p(1 - qx)^{-3}$

(a) find the value of p and the value of q.

(2)

(b) Expand $\dfrac{1}{(2-x)^3}$ in ascending powers of x up to and including the term in x^3 and express each coefficient as an exact fraction in its lowest terms.

(3)

$$\mathrm{f}(x) = \frac{a + bx}{(2-x)^3} \quad \text{where } a \text{ and } b \text{ are integers}$$

The first three terms of the expansion of $\mathrm{f}(x)$ are $\dfrac{3}{8} - \dfrac{43}{16}x + cx^2$

(c) Find the value of a and the value of b.

(3)

(d) Find the exact value of c.

(2)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 10 marks)

10 The equation of a curve C is $y = \mathrm{f}(x)$ where $\mathrm{f}'(x) = 3x^2 - 4x - p$ and $p \neq 0$

The points with coordinates (2, 0) and (−1, 9) lie on C.

(a) Show that C has equation $y = x^3 - 2x^2 - 4x + 8$

(6)

The straight line l has equation $y = 8 - 4x$

(b) Use algebraic integration to find the exact area of the finite region bounded by C and l.

(6)

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 12 marks)

11 The curve C has equation $y = \dfrac{3x - 2}{x + 1}$

(a) Write down an equation of the asymptote to C which is parallel to the

(i) x-axis (ii) y-axis

(2)

(b) Find the coordinates of the point where C crosses the

(i) x-axis (ii) y-axis

(2)

(c) Sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

The straight line l has equation $y = mx + 4$

Given that there are **no** points of intersection between l and C,

(d) show algebraically that the range of possible values of m can be written as

$$a - 2\sqrt{b} < m < a + 2\sqrt{b}$$

where a and b are integers whose values need to be found.

(7)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS