Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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**Pearson Edexcel International GCSE**

Centre Number | | | | | |

Candidate Number | | | | |

# Monday 23 November 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02**

# Further Pure Mathematics

## Paper 2

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

**International GCSE in Further Pure Mathematics Formulae sheet**

## Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

## Series

### Arithmetic series

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### Geometric series

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

### Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

## Calculus

### Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

## Trigonometry

### Cosine rule

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

## Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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**Answer all TEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** A particle $P$ is moving in a straight line. At time $t$ seconds, $t \geqslant 0$, the displacement, $s$ metres, of $P$ from a fixed point $O$ of the line is given by

$$s = 3 + 8t + t^2 - \frac{1}{3}t^3$$

Find the distance of $P$ from $O$ when $P$ is instantaneously at rest.

**(4)**

**(Total for Question 1 is 4 marks)**

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**2** The region enclosed by the curve with equation $y = e^{3x}$, the $x$-axis, the $y$-axis and the line with equation $x = 3$ is rotated through 360° about the $x$-axis.

Use algebraic integration to find, in terms of $\pi$ and e, the volume of the solid generated.

**(4)**

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**Question 2 continued**

**(Total for Question 2 is 4 marks)**

**3** (a) Expand

$$(1 + px)^{-5} \quad p \neq 0$$

in ascending powers of $x$, up to and including the term in $x^4$
Give each term in its simplest form.

**(3)**

The coefficient of $x^r$ in the expansion is $c_r$

Given that $c_4 = 2c_3$

(b) find the value of $p$.

**(2)**

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**Question 3 continued**

**(Total for Question 3 is 5 marks)**

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**4** (i) Solve the equation $16\log_r 4 = \log_4 r$

**(2)**

(ii) Solve the equation $\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = 1 + \log_5 x + \log_5 x^2$

**(5)**

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**Question 4 continued**

**(Total for Question 4 is 7 marks)**

**5** (a) Show that $\sum_{r=1}^{n}(3r+5) = \frac{1}{2}n(3n+13)$

**(3)**

(b) Hence evaluate $\sum_{r=35}^{50}(3r+5)$

**(2)**

Given that $\sum_{r=1}^{n}(3r+5) = 385$

(c) find the value of $n$.

**(3)**

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**Question 5 continued**

**(Total for Question 5 is 8 marks)**

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**6** $f(x) = 4x^2 - 3x - 5$

The equation $f(x) = 0$ has roots $\alpha$ and $\beta$

Without solving the equation $f(x) = 0$

(a) form an equation, with integer coefficients, that has roots $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$

**(6)**

$g(x) = 4x^2 + px + q$ where $p$ and $q$ are constants

Given that the equation $g(x) = 0$ has roots $3\alpha + \beta$ and $\alpha + 3\beta$

(b) find the value of $p$ and the value of $q$

**(5)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 11 marks)**

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**7** A geometric series has first term $(x - 3)$, second term $(x + 1)$ and third term $(4x - 2)$.

(a) Find the two possible values of $x$.

**(5)**

Given that $x < 1$,

(b) show that the series is convergent.

**(2)**

The sum to infinity of the series is $S$.

(c) Find the value of $S$.

**(2)**

The sum of the first $n$ terms of the series is $S_n$

Given that $\frac{S}{S_n} = \frac{256}{255}$

(d) find the value of $n$.

**(3)**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 12 marks)**

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**8** The curve $C_1$ has equation $y = 5e^{-2x} + 4$

The curve $C_2$ has equation $y = e^{2x}$

The curves $C_1$ and $C_2$ intersect at the point $A$.

(a) Find the exact coordinates of $A$.

**(4)**

The tangent at $A$ to $C_1$ intersects the $x$-axis at the point $B$.

(b) Show that the $x$ coordinate of $B$ is $\frac{1}{2}(5 + \ln 5)$

**(5)**

The tangent at $A$ to $C_2$ intersects the $x$-axis at the point $D$.

(c) Find the area of $\Delta ABD$.

**(6)**

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 15 marks)**

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**9** A curve $C$ has equation

$$y = \frac{2 + 4x - x^2}{2x + 1} \qquad x \neq -\frac{1}{2}$$

(a) Write the equation of $C$ in the form $ax^2 + (by - 4)x + (y - c) = 0$, where $a$, $b$ and $c$ are integers whose values are to be found.

**(3)**

(b) Hence show that $x$ is real when $y \leqslant 2$ and when $y \geqslant 3$

**(4)**

(c) Find the coordinates of the stationary points on $C$.

**(6)**

(d) Sketch $C$ showing clearly

(i) the exact coordinates of the points where $C$ crosses the $x$-axis,

(ii) the asymptote to $C$ that is parallel to the $y$-axis,

(iii) the coordinates of the stationary points.

**(5)**

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 18 marks)**

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**10** (a) Show that

$$\cos(A + B) + \cos(A - B) = 2\cos A \cos B$$

**(2)**

(b) Hence show that

$$\cos P + \cos Q = 2\cos\frac{P + Q}{2}\cos\frac{P - Q}{2}$$

**(3)**

(c) Solve, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$, the equation

$$\cos 5\theta + \cos 7\theta = 0$$

Give each solution in terms of $\pi$

**(4)**

(d) Show that

$$\cos 8x + 2\cos 6x + \cos 4x = 4\cos 6x \cos^2 x$$

**(3)**

(e) Use calculus to find the exact value of

$$\int_0^{\frac{\pi}{3}} \cos 6x \cos^2 x \, dx$$

**(4)**

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**Question 10 continued**

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**Question 10 continued**

**Question 10 continued**

**(Total for Question 10 is 16 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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