Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Thursday 19 November 2020

Afternoon (Time: 2 hours)	Paper Reference **4PM1/01R**

Further Pure Mathematics

Paper 1R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Answer all TWELVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here is a formula

$$P = 3 + 2\sin\left(\frac{3\pi t}{8}\right) \qquad 0 \leqslant t \leqslant 12$$

(a) Find the exact value of P when $t = \dfrac{10}{3}$

(2)

(b) Find

(i) the largest value of P

(ii) the smallest value of P

(2)

(c) Find the least value of t for which $P = 4$

(3)

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2 (a) Express $x^2 + 4x - 8$ in the form $(x + a)^2 + b$ where a and b are constants whose values are to be found.

(2)

(b) Use algebra to solve the simultaneous equations

$$y = x^2 + 4x - 8$$
$$y = 2x + 7$$

(5)

The curve C has equation $y = x^2 + 4x - 8$
The straight line L has equation $y = 2x + 7$

Using the same axes and the results of part (a) and part (b),

(c) sketch C and L, showing clearly the coordinates of the turning point of C and the coordinates of the points of intersection of C and L.

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

Question 2 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 11 marks)

P 6 2 2 8 3 A 0 9 4 0

3 The *n*th term of an arithmetic series is u_n such that

$$u_n = \ln a + (n - 1)\ln b$$

where a and b are positive integers.

Given that $u_2 = \ln 12$ and that $u_5 = \ln 768$

find the value of a and the value of b.

(7)

Question 3 continued

(Total for Question 3 is 7 marks)

4 The curve C has equation

$$y = x^3 - 3x^2 - 24x + 6$$

(a) Use calculus to find the coordinates of each of the stationary points on C.

(4)

(b) Determine the nature of each of these stationary points.
Justify your answers.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 6 marks)

5 (a) Expand $\sqrt{1-x}$ in ascending powers of x up to and including the term in x^3

Give each coefficient as an exact fraction in its lowest terms.

(3)

(b) Using your expansion with a suitable value of x, obtain an approximation, to 6 decimal places, of $\sqrt{0.92}$

(3)

(c) Hence find an approximation, to 5 decimal places, of $\sqrt{23}$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 8 marks)

6 (a) Show that

$$\sin(A + B) + \sin(A - B) = 2\sin A \cos B$$

(2)

(b) Hence express $2\sin 7x \cos x$ in the form $\sin mx + \sin nx$ where m and n are integers, giving the value of m and the value of n.

(1)

(c) Use calculus to evaluate

$$\int_0^{\frac{\pi}{4}} (6\sin 7x \cos x)\,\mathrm{d}x$$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

Question 6 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 7 marks)

Turn over

7 The length of each side of a cube S_1 is increasing at a constant rate of 0.1 m/s.

(a) Find, in m^3/s, the rate of increase of the volume of the cube S_1 when the length of each side of the cube is 2 m.

(4)

The total surface area of a different cube S_2 is increasing at a constant rate of $0.05\,m^2/s$.

(b) Find in m^3/s, the rate of increase of the volume of the cube S_2 when the length of each side of the cube is 6 m.

(5)

P 6 2 2 8 3 A 0 2 0 4 0

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8

$$f(x) = 3x^2 - x + 4$$

$$g(x) = x^2 - px + q$$

The roots of the quadratic equation $f(x) = 0$ are α and β

The roots of the quadratic equation $g(x) = 0$ are $\left(\alpha + \frac{1}{\alpha}\right)$ and $\left(\beta + \frac{1}{\beta}\right)$

Without solving the equation $f(x) = 0$

(a) show that $p = \frac{7}{12}$

(3)

(b) Find the value of q

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9 Showing your working clearly, use algebra to solve the equations

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$

(7)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 7 marks)

10 (a) Solve the equation

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \text{for } 0 \leqslant x \leqslant 2\pi$$

Give your solutions in terms of π, where appropriate.

(3)

(b) Solve the equation

$$3\sin\theta + 5\cos\theta = 0 \qquad \text{for } -360° \leqslant \theta \leqslant 360°$$

Give your solutions to the nearest degree.

(3)

(c) Solve the equation

$$1 + \sin 2y = 2\cos^2 2y \qquad \text{for } -180° \leqslant y \leqslant 0°$$

(5)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 11 marks)

11

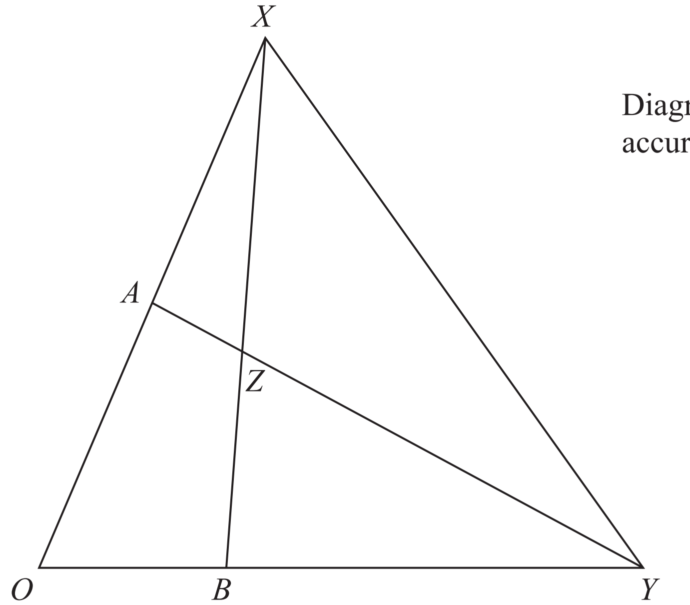

Figure 1

Figure 1 shows a triangle OXY

$$\overrightarrow{OX} = 2\mathbf{a} \text{ and } \overrightarrow{OY} = 3\mathbf{b}$$

A is the midpoint of OX and B is the point on OY such that $OB : BY = 1 : 2$
The lines XB and AY intersect at Z.

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of **a** and **b**

(1)

(b) Using a vector method, find $\overrightarrow{OZ}$ as a simplified expression in terms of **a** and **b**

(9)

The point M on XY is such that O, Z and M are collinear.

(c) Find $\overrightarrow{OM}$ as a simplified expression in terms of **a** and **b**

(3)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 13 marks)

12

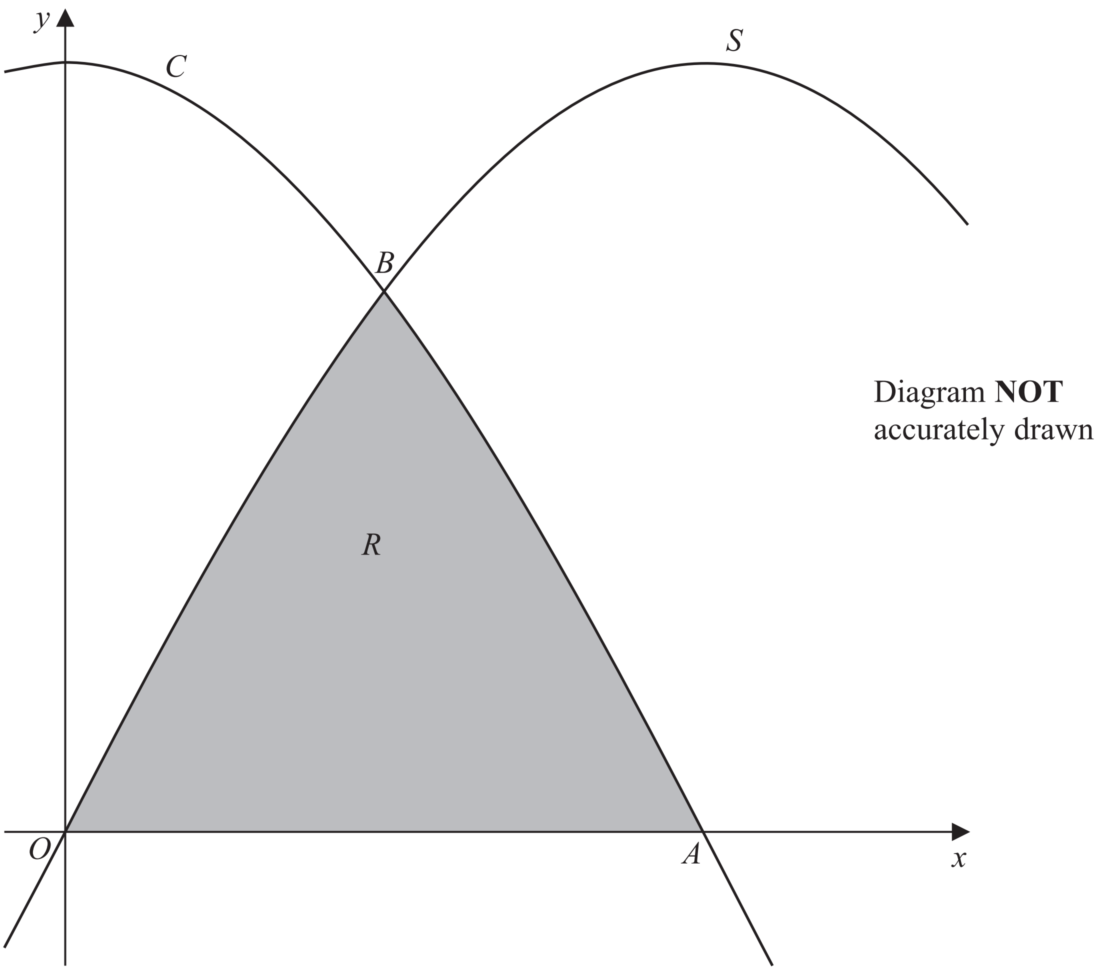

Figure 2

The region R, shown shaded in Figure 2, is bounded by the x-axis, the curve S with equation $y = 2\sin x$ and the curve C with equation $y = 2\cos x$.
As shown in Figure 2, C crosses the x-axis at the point A.

(a) Write down the x coordinate of A.

(1)

As shown in Figure 2, C and S intersect at the point B.

(b) Find the x coordinate of B.

(2)

(c) Using calculus, find the area of the shaded region R.
Give your answer in the form $a - \sqrt{b}$ where a and b are integers.

(4)

Question 12 continued

Question 12 continued

(Total for Question 12 is 7 marks)

TOTAL FOR PAPER IS 100 MARKS