Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 23 November 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

Further Pure Mathematics

Paper 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The nth term of an arithmetic series A is a_n

The nth term of a geometric series G is t_n

For these two series

$$a_1 = t_1 \qquad a_{10} = t_3 = 48 \qquad a_{10} = 4t_2$$

Find

(i) the common ratio of G,

(ii) the common difference of A.

(6)

(Total for Question 1 is 6 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

2 $\quad f(x) = x^3 + px + q \qquad$ where p and q are constants.

The remainder when $f(x)$ is divided by $(x - 1)$ is -12

The remainder when $f(x)$ is divided by $(x - 4)$ is 30

(a) Find the value of p and the value of q.

(6)

Using your values of p and q

(b) show that $f(3) = 0$

(1)

(c) Express $f(x)$ as a product of linear factors.

(3)

(d) Hence solve the equation $f(x) = 0$

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 11 marks)

3

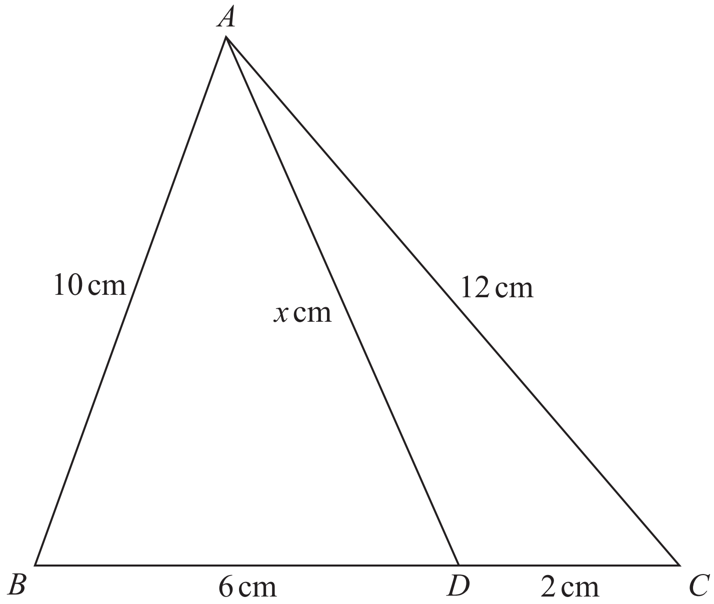


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the triangle ABC in which $AB = 10$ cm and $AC = 12$ cm.
The point D lies on BC such that $BD = 6$ cm, $DC = 2$ cm and $AD = x$ cm.

(a) Show that $x = 11$

(4)

(b) Find the area, in cm^2 to 3 significant figures, of triangle ADB.

(4)

P 6 2 2 8 4 A 0 6 3 2

Question 3 continued

(Total for Question 3 is 8 marks)

4 (a) Complete the table of values for $y = 2x + 1 + \frac{2}{x^2}$

Give your answers to 2 decimal places where appropriate.

x	0.5	1	1.5	2	2.5	3	3.5
y		5			6.32		8.16

(2)

(b) On the grid opposite, draw the graph of $y = 2x + 1 + \frac{2}{x^2}$ for $0.5 \leqslant x \leqslant 3.5$

(2)

(c) Use your graph to obtain estimates, to 1 decimal place, of the roots of the equation

$$2x + \frac{2}{x^2} = 7 \quad \text{in the interval } 0.5 \leqslant x \leqslant 3.5$$

(2)

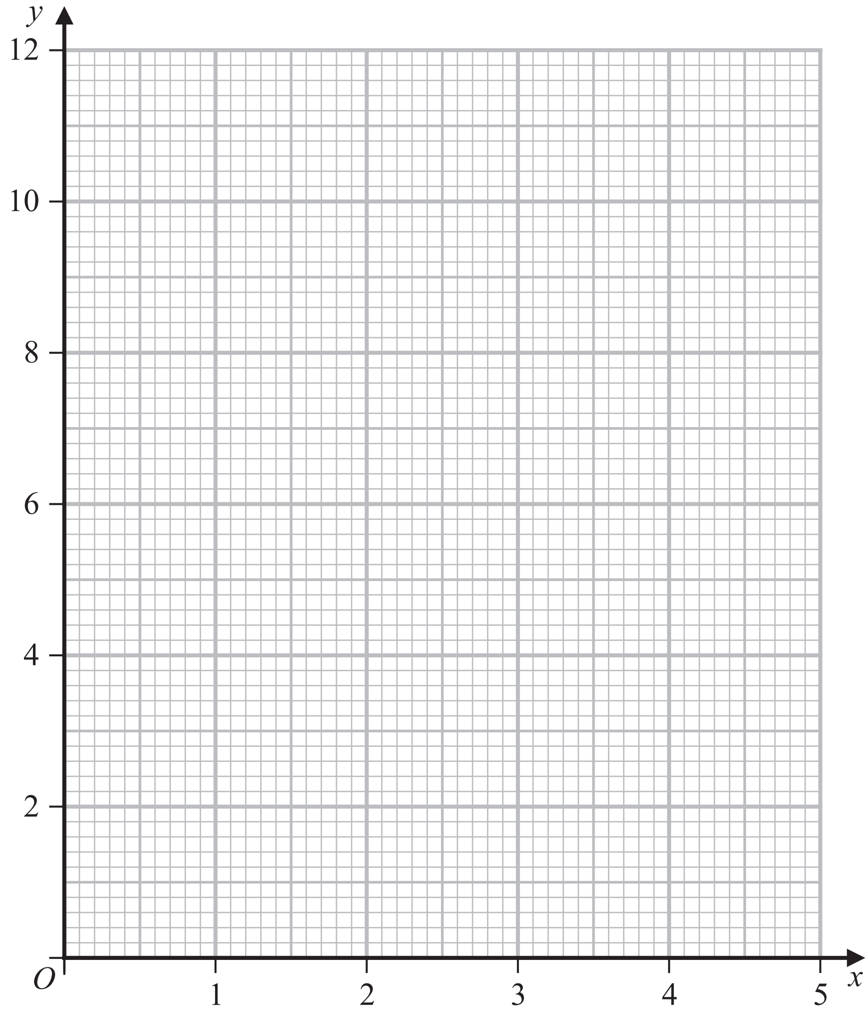
(d) By drawing a suitable straight line on the grid, obtain estimates, to 1 decimal place, of the roots of the equation

$$\frac{3x}{2} + \frac{2}{x^2} = 5 \quad \text{in the interval } 0.5 \leqslant x \leqslant 3.5$$

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

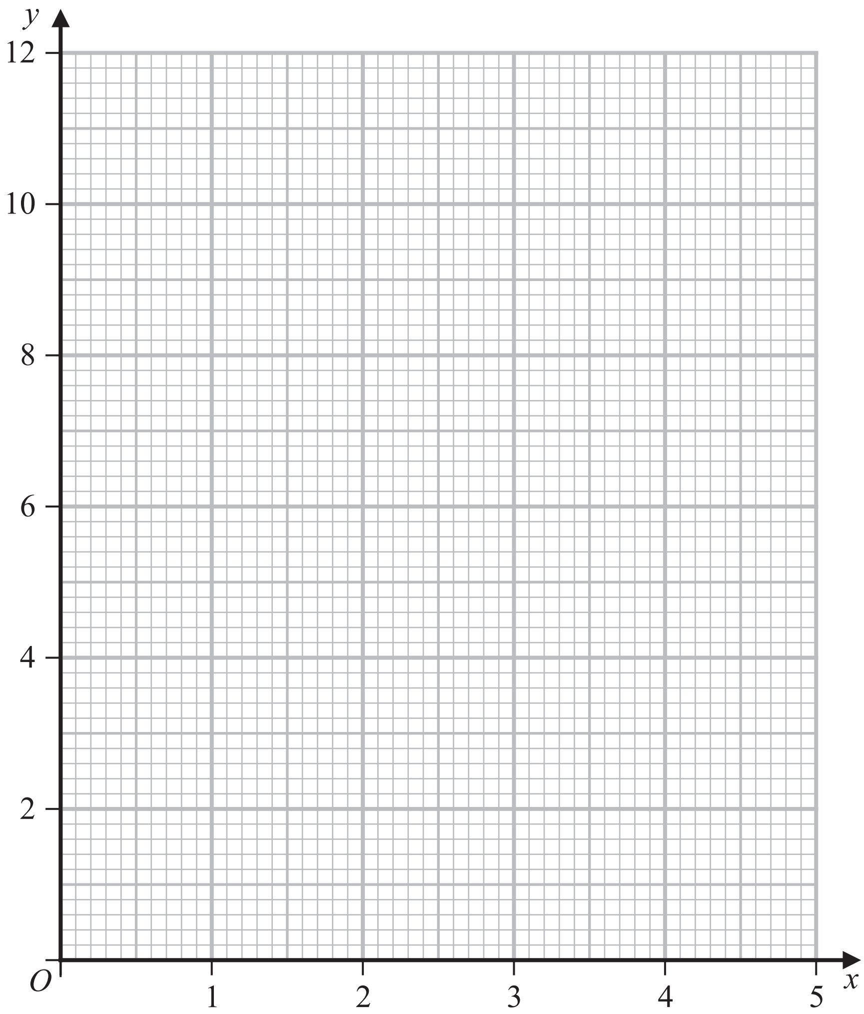


Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

Question 4 continued

Only use this grid if you need to redraw your graph.



(Total for Question 4 is 11 marks)

5

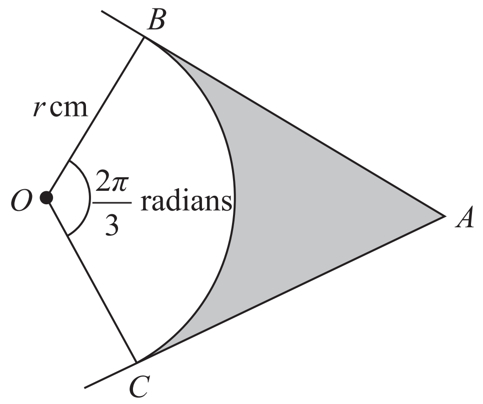


Diagram **NOT** accurately drawn

Figure 2

In Figure 2, AB and AC are tangents to a circle with centre O and radius r cm.

The points B and C lie on the circle so that OBC is a sector of this circle and $\angle BOC = \frac{2\pi}{3}$ radians.

Given that the area of the shaded region is $10\,\text{cm}^2$,

find, to 3 significant figures, the value of r.

(8)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 8 marks)

6

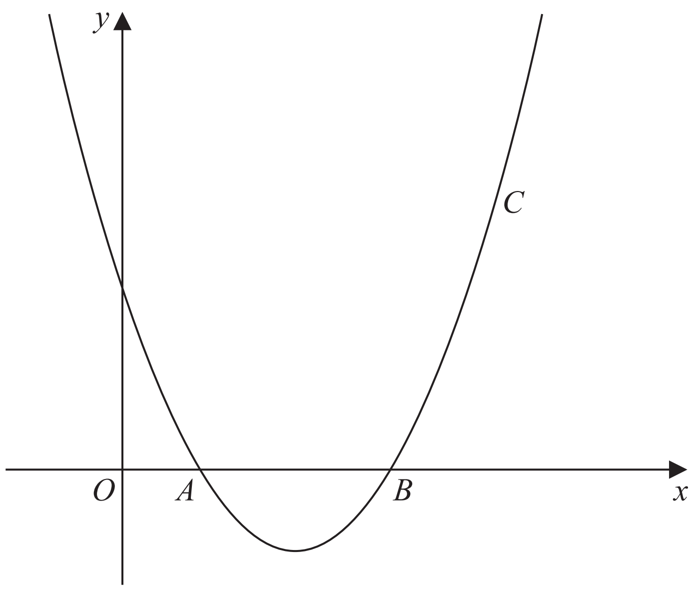


Diagram **NOT** accurately drawn

Figure 3

The curve C with equation $y = x^2 - 5x + 4$ crosses the x-axis at the points A and B, as shown in Figure 3

(a) Find the coordinates of A and the coordinates of B.

(3)

The tangent to C at A meets the tangent to C at B at the point T.

(b) Find the coordinates of T.

(6)

The normal to C at A meets the normal to C at B at the point N.

(c) Find the coordinates of N.

(3)

(d) Find the area of the quadrilateral $ATBN$.

(3)

Question 6 continued

Question 6 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 15 marks)

P 6 2 2 8 4 A 0 1 7 3 2

7 (a) Find the set of values of k for which the equation $kx^2 - 4x + 2k = 7$ has real roots

(4)

Given that the roots of the equation $kx^2 - 4x + 2k = 7$ are α and β,

(b) form a quadratic equation with roots $\frac{\alpha + 1}{\alpha}$ and $\frac{\beta + 1}{\beta}$

Give each coefficient in terms of k.

(8)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8 Solve the equation $\log_3 x - 2\log_x 3 = 1$

(7)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 7 marks)

P 6 2 2 8 4 A 0 2 3 3 2

9 Given that

$$x = e^{-t} \sin 2t$$

show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$$

(8)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10

$$f(x) = 32x^3 - 33x + 1$$

(a) Show that $f(1) = 0$

(1)

(b) Hence using an algebraic method solve $f(x) = 0$

(4)

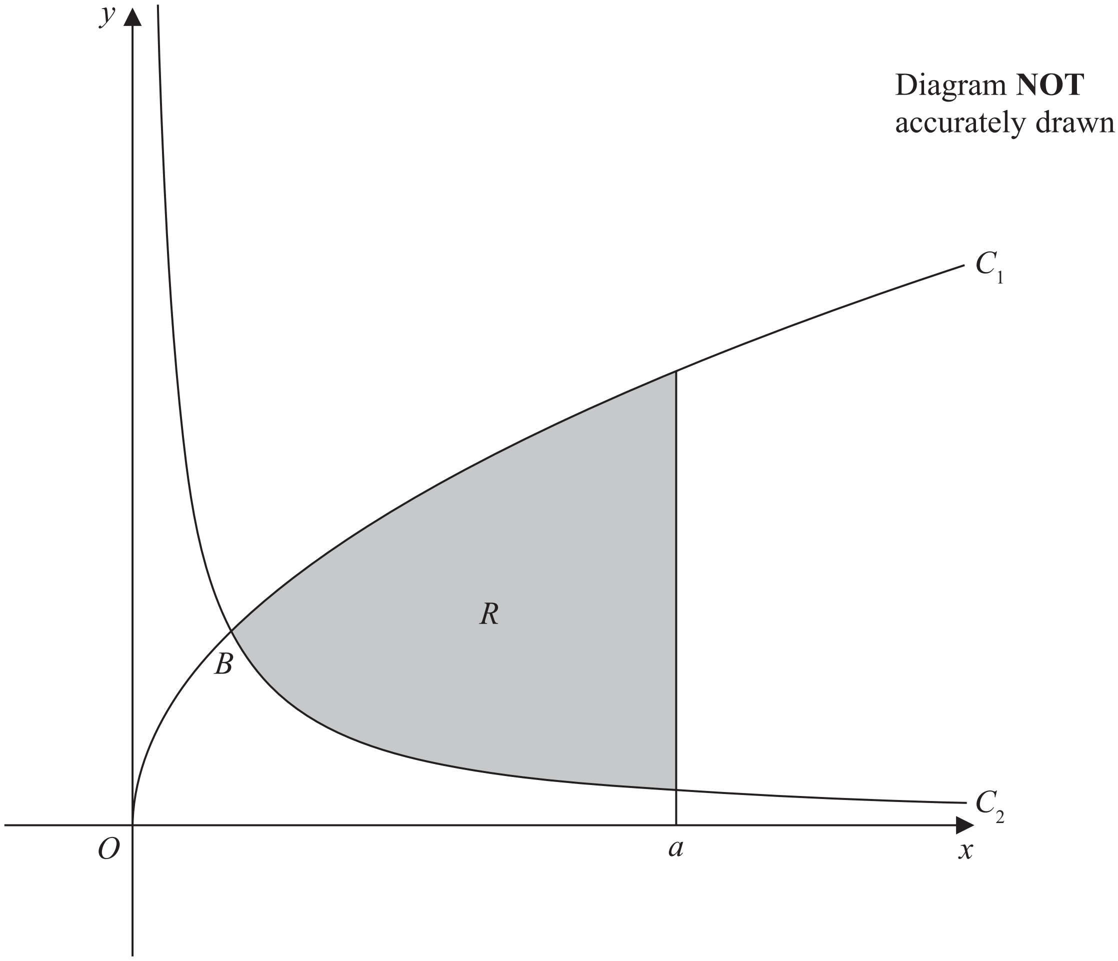


Figure 4

The region R, shown shaded in Figure 4, is bounded by the curve C_1 with equation $y = \sqrt{x}$, by the curve C_2 with equation $y = \dfrac{1}{8x}$ and by the line with equation $x = a$

The curves C_1 and C_2 intersect at the point B, with x coordinate p, where $p < a$

(c) Find the value of p.

(2)

The region R is rotated through 360° about the x-axis to generate a solid with volume $\dfrac{27\pi}{64}$

(d) Use algebraic integration to find the value of a.

(7)

Question 10 continued

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS